

Overview of course modules, homework assignments, and exams

1. Required textbook

Introduction to Probability, Statistics, and Random Processes, Hossein Pishro-Nik, Kappa Research, LLC. August 24, 2014. ISBN-10: 0990637204. ISBN-13: 978-0990637202. Please note that the textbook is also freely available online: <https://www.probabilitycourse.com/>

“Textbook videos” are provided by the authors of the textbook and embedded in the upper right corner of the online text of each chapter. These textbook videos are not the same as the lecture videos, which are given by the course’s instructor. The recommended sequence of using these videos is to start with the textbook videos and then going to the lecture videos.

2. Overview of course modules, homework assignments, and exams

- Module 1: Randomness and probability axioms.
 - Lectures 1 – 3.
 - Homework 1.
- Module 2: Conditional probability, law of total probability, Bayes rule.
 - Lectures 4 – 6.
 - Homework 2.
- Module 3: Combinatorics
 - Lectures 7 – 8.
- Midterm I: covering all materials in Modules 1 & 2.
- Module 4: Random variables, discrete distributions.
 - Lectures 9 – 11.
 - Homework 3: covering modules 3 & 4.
- Module 5: Cumulative distribution function, functions of RV.
 - Lecture 12 – 13.
 - Homework 4.
- Midterm II: covering all materials in Modules 3 – 5.
- Module 6: Continuous RV, uniform and normal distributions, law of large numbers.
 - Lectures: 14 – 16.
- Module 7: Hypothesis testing.
 - Lectures 17 – 18.
 - Homework 5: covering modules 6 & 7.
- Final: covering all materials in Modules 1 – 7.

3. Module details

Module 1: Randomness and probability axioms

Lecture 1. Randomness and set theory

- Reading materials: Pishro-Nik chapter 1.0 – 1.2.2; Handout: Throwing a dart; Handout: Computer simulation challenge; Handout: The π challenge.
- Core concepts:
 - Randomness
 - Interpretation of probability: frequency vs. subjective personal belief
 - Set theory and set operations
 - Venn diagram
- Not required contents: Pishro-Nik chapter 1.2.2: Cartesian product.

Lecture 2. Functions and probability axioms

- Reading materials: Pishro-Nik chapter 1.2.3 – 1.3.2; Handout: Computer simulation challenge; Handout: The π challenge.
- Textbook videos: Video 1.2 – Sets, Functions; Video 1.3 – Probability Axioms.
- Core concepts:
 - Cardinality and finite sets
 - Inclusion-exclusion principle
 - Functions: input, output, the unique output of any input, domain, co-domain, range.
 - Random experiments: outcome, sample space, event.
 - Union and intersection of two events.
 - Axioms of probability
- Not required contents: Pishro-Nik Theorem 1.4, Theorem 1.5.

Lecture 3. Calculating probabilities

- Reading materials: Pishro-Nik chapter 1.3.2 – 1.3.4; Handout: The Monty Hall problem.
- Textbook videos: Video 1.4 – Discrete and Continuous Probability Models.
- Important concepts:
 - Discrete probability model

Homework 1, covering module 1.

Module 2: Conditional probability, law of total probability, Bayes rule

Lecture 4. Conditional probability and independence

- Reading materials: Pishro-Nik chapter 1.4 – 1.4.1; Handout: Proof of the chain rule of conditional probability.
- Textbook videos: Video 1.5 – Conditional Probability; Video 1.6 – Independent Events – Part 1; Video 1.7 – Independent Events – Part 2.
- Core concepts:
 - Conditional probability
 - Chain rule of conditional probability
 - Independence

- The difference between independence and disjoint
- Not required contents: Prior probability; The proof of Lemma 1.1.

Lecture 5. Law of total probability

- Reading materials: Pishro-Nik chapter 1.4.2; Handout: The Monty Hall Problem.
- Textbook videos: Video 1.8 – Law of Total Probability.
- Core concepts:
 - Law of Total Probability

Lecture 6. Bayes rule

- Reading materials: Pishro-Nik chapter 1.4.3.
- Textbook videos: Video 1.9 – Bayes Theorem, Video 1.10 – Boy or Girl Paradox.
- Core concepts:
 - Bayes rule

Homework 2, covering module 2.

Module 3: Combinatorics

Lecture 7. Sampling, permutations, and combinations

- Reading materials: Pishro-Nik chapter 2.1 – 2.1.2.
- Textbook videos: Video 2.1 – Counting – Part 1; Video 2.2 – Counting – Part 2.
- Core concepts:
 - Ordered sampling with replacement
 - Permutations: Ordered sampling without replacement

Lecture 8. Combinations, binomial distribution.

- Reading materials: Pishro-Nik chapter 2.1.3.
- Textbook videos: Video 2.3 - Counting - Part 3: K-Combinations and the Binomial Formula.
- Core concepts:
 - Combinations: Unordered sampling without replacement
 - Bernoulli Trials
 - Binomial Distribution
- Not required contents: Pishro-Nik chapter 2.1.4: Unordered sampling with replacement.

Midterm I, covering all materials in Modules 1 and 2.

Module 4: Random variables and discrete distributions

Lecture 9. Random variables

- Reading materials: Pishro-Nik chapter 3.1 – 3.1.4.
- Textbook videos: Video 3.1 – Introduction to Random Variables: Discrete Random Variables - Part 1; Video 3.2 - Discrete Random Variables, PMF, Independent Random Variables.
- Core concepts:
 - Random variable (RV)
 - Discrete random variable
 - Probability mass function (PMF)
 - Independent random variables

Lecture 10. Bernoulli, Geometric, and binomial distributions.

- Reading materials: Pishro-Nik chapter 3.1.5.
- Textbook videos: Video 3.3 – Geometric and Binomial Random Variables.
- Core concepts:
 - Bernoulli distribution
 - Geometric distribution
 - Binomial distribution
- Not required contents: Pishro-Nik chapter 2.1.5: Negative Binomial distribution.

Lecture 11. Hypergeometric distribution and applications to genetic data

- Reading materials: Pishro-Nik chapter 3.1.5; Handout: Applications of the hypergeometric distribution: Color of cards; Patients in a clinic; Handout: Genotype vs. phenotype
- Textbook videos: Video 3.4 – Poisson, Pascal, and Hypergeometric Distributions
- Core concepts:
 - Hypergeometric distribution
 - Genotype
 - Phenotype
 - Single nucleotide polymorphism (SNP)
- Not required contents: Pishro-Nik chapter 2.1.5: Poisson distribution.

Homework 3, covering modules 3 and 4.

Module 5: Cumulative distribution function, functions of RV, continuous RV.

Lecture 12. Cumulative distribution function, Expectation

- Reading materials: Pishro-Nik chapter 3.2.1 – 3.2.2.
- Textbook videos: Video 3.5 – CDF for Discrete Random Variables, Video 3.6 – Expectation of Discrete Random Variables.
- Core concepts:

- Cumulative distribution function (CDF)
- Expectation
- Not required contents: Pishro-Nik chapter 3.2.2: Example 3.13.

Lecture 13. Functions of random variables, Variance

- Reading materials: Pishro-Nik chapter 3.2.3 – 3.2.4.
- Textbook videos: Video 3.8 – Functions of Discrete Random Variables, Video 3.9 - Variance and Standard Deviation.
- Core concepts:
 - Function of RV
 - Expected value of the function of a RV
 - Variance
 - Variance of the sum of independent RVs

Homework 4, covering module 5.

Midterm II. Covering all materials in Modules 3 – 5.

Module 6: Continuous RV, uniform and normal distributions, law of large numbers

Lecture 14. Functions of random variables, Variance

- Reading materials: Pishro-Nik chapter 4.1.0 – 4.1.2.
- Textbook videos: Video 4.2 – Probability Density Function (PDF) for Continuous Random Variables; Video 4.3 - Expected Values for Continuous Random Variables
- Core concepts:
 - Probability density function (PDF)
 - Expectation and variance
 - Law of the unconscious statistician (LOTUS)
- Not required contents: Pishro-Nik chapter 4.1.3.

Lecture 15. Uniform distribution, Normal distribution

- Reading materials: Pishro-Nik chapter 4.2.1, 4.2.3.
- Textbook videos: Video 4.7 – Uniform Distribution, Video 4.9 – Normal distribution.
- Prior knowledge from calculus:
 - Gaussian integral (https://en.wikipedia.org/wiki/Gaussian_integral),

$$\int_{-\infty}^{\infty} e^{-x^2} dx = \sqrt{\pi}.$$

- Integration by parts (https://en.wikipedia.org/wiki/Integration_by_parts),

$$\int_a^b u(x)v'(x)dx = [u(x)v(x)]_a^b - \int_a^b u'(x)v(x)dx$$

$$= u(b)v(b) - u(a)v(a) - \int_a^b u'(x)v(x)dx.$$

- Core concepts:
 - Uniform distribution
 - Standard normal RV
 - The Φ function (CDF of standard normal)
 - Normal RVs
- Not required contents: Pishro-Nik chapter 4.2.2. Exponential distribution, 4.2.4. Gamma distribution, 4.3. Mixed RVs, 5. Joint distributions, 6. Multiple RVs.

Lecture 16. Law of large numbers

- Reading materials: Pishro-Nik chapter 7.1.1 – 7.1.2.
- Textbook videos: Video 3.8 – Functions of Discrete Random Variables, Video 3.9 - Variance and Standard Deviation.
- Core concepts:
 - Sample mean
 - The weak law of large numbers
 - Central limit theorem
- Not required contents: Pishro-Nik chapter 7.1.1: Proof of the weak law of large numbers.

Module 7: Hypothesis testing

Lecture 17. Hypothesis testing

- Reading materials: Pishro-Nik chapter 8.4.1-8.4.2, chapter 8.4.4. Handout: The instructor's cheat sheet to hypothesis testing.
- Core concepts:
 - Null and alternative hypotheses
 - Test statistic
 - P-value
- Not required contents: Pishro-Nik chapters 8.1 – 8.3, 8.4.3.

Lecture 18. Acceptance and rejection regions

- Reading materials: Pishro-Nik chapters 8.4.2 and 8.4.4. Handout: The instructor's cheat sheet to hypothesis testing.
- Core concept:
 - Acceptance region and rejection region
 - Type I error

- Significance level
- Type II error
- Not required contents: Pishro-Nik chapter 8.4.5.

Homework 5, covering Modules 6 and 7.

Final exam, covering all materials in Modules 1 – 7.